

**International Master's Programs of Chemical Engineering in the Graduate School of Engineering,
Kyushu University (Academic Year from October, 2026)**

Subject : Fluid Dynamics

$$(1.1) \quad \tau_{xz}(x) = \frac{p_0 - p_L}{L} x, \quad v_z(x) = \frac{(p_0 - p_L)B^2}{2\mu L} \left[1 - \left(\frac{x}{B} \right)^2 \right]$$

$$(1.2) \quad v_{z,max} = \frac{(p_0 - p_L)B^2}{2\mu L}$$

$$(1.3) \quad \frac{\langle v_z \rangle}{v_{z,max}} = \frac{2}{3}$$

$$(1.4) \quad W = \frac{2(p_0 - p_L)\rho B^3 W}{3\mu L}$$

(1.5) The assumption $B \ll W$ means that the plate spacing is much smaller than the slit width. Therefore, the effects of the side edges can be neglected, and the velocity can be regarded as a function of x only: $v_z = v_z(x)$.

The assumption $W \ll L$ means that the flow length is sufficiently long compared with the slit width. Therefore, entrance and exit effects can be neglected, and the flow can be treated as fully developed.

If B becomes comparable to W , edge effects can no longer be neglected. In that case, the velocity depends not only on x , but also on the width direction: $v_z = v_z(x, y)$.

$$(2.1) \quad u = \frac{\partial \phi}{\partial z}, \quad v = \frac{\partial \phi}{\partial r}$$

$$(2.2) \quad u = -\frac{1}{r} \frac{\partial \Psi}{\partial r}, \quad v = \frac{1}{r} \frac{\partial \Psi}{\partial z}$$

$$(2.3) \quad \frac{1}{r} \frac{\partial}{\partial r} (\rho r v) + \frac{\partial}{\partial z} (\rho u) = \frac{1}{r} \frac{\partial}{\partial r} \left(\rho \frac{\partial \Psi}{\partial z} \right) - \frac{1}{r} \frac{\partial}{\partial z} \left(\rho \frac{\partial \Psi}{\partial r} \right) = 0$$

$$(2.4) \quad \frac{dz}{u} = \frac{dr}{v}, \quad \frac{dz}{dr} = \frac{u}{v}, \quad v dz = u dr$$

$$d\Psi = \frac{\partial \Psi}{\partial z} dz + \frac{\partial \Psi}{\partial r} dr = v r dz - u r dr = 0$$