**International Master's Programs of Chemical Engineering in the Graduate School of Engineering, Kyushu University (Academic Year from October, 2026)**

Subject： Fluid Dynamics（1 sheet）

1. (30 points)

A Newtonian fluid with density $\rho$ and viscosity $\mu$ flows steadily in laminar flow through a narrow slit formed by two parallel plates separated by a distance $2B$, as shown in Fig. 1. The width of the slit is $W$, and the length in the flow direction is $L$, where $B \ll W \ll L$.

Assume that edge effects can be neglected and that the flow is fully developed.

The fluid enters at $z = 0$ with pressure $p_0$ and exits at $z = L$ with pressure $p_L$, where $p_0 > p_L$.

The effect of gravity can be neglected. Assume that the only nonzero velocity component is $v_z(x)$. Define any necessary physical properties and parameters. Answer the following questions.

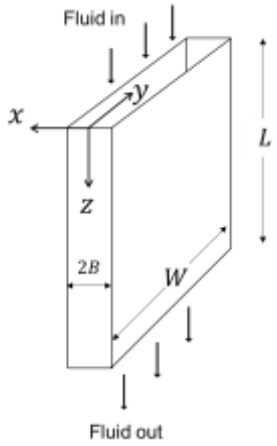

Fig. 1

(1.1) Consider a fluid shell of thickness d$x$ in the $x$-direction. Derive the momentum flux $\tau_{xz}(x)$ and the velocity distribution $v_z(x)$ from the momentum balance in the $z$-direction.

(1.2) Determine the maximum velocity $v_{z,max}$.

(1.3) Determine the ratio of the average velocity $\langle v_z \rangle$ to the maximum velocity $v_{z,max}$.

(1.4) Determine the total mass flow rate $w$ through the slit cross section.

(1.5) Explain the physical meaning of the assumption, $B \ll W \ll L$, used in the above analysis. Also explain why it is no longer appropriate to directly use the velocity distribution and the mass flow rate obtained above if $B$ becomes comparable to $W$.

2. (20 points)

Answer the following questions regarding a fluid of density $\rho$ undergoing steady axisymmetric two-dimensional motion. Let $u$ be the velocity in the axial direction and $v$ be the velocity in the radial direction in cylindrical coordinates.

(2.1) How is the velocity potential $\phi$ of an incompressible flow defined?

(2.2) How is the stream function $\Psi$ of an incompressible flow defined?

(2.3) Show that the stream function defined in (2.2) satisfies the continuity equation.

(2.4) Show that a line where the stream function takes as constant value is a streamline.